Metaplectic modular categories and the associated TQFT

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Overview

Modular tensor categories: definition and conventions

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Notations:

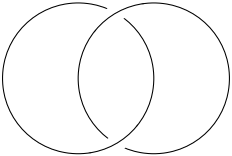
$\mathcal{I}$ = the set of representatives of isomorphism classes (with a special object $\underline{1}$ representing the unit),

d_i = quantum dimension of the object $i \in \mathcal{I}$,

$D^2 = \sum_{i \in \mathcal{I}} d_i^2$.

Modular tensor categories: definition and conventions

The S-matrix is defined to be an $|\mathcal{I}| \times |\mathcal{I}|$-complex matrix whose (i,j)-th entry is given by:

$S_{i,j} = \frac{1}{D}$ i  j , $i, \ j \in \mathcal{I}.$ (1)

TQFT and the spaces V_g

3-dimensional TQFT: a tensor functor $\mathcal{V} : Cob^3 \to Vec_{\mathbb{C}}$. Given any MTC, there exists a TQFT.

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$\forall \vec{i} = (i_1, ..., i_g) \in \mathcal{I}^g$, let

$$V_g^{\vec{i}} := \mathrm{Hom}(\underline{1},\ i_1 \otimes i_1^* \otimes \cdots \otimes i_g \otimes i_g^*), \tag{2}$$

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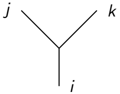
then V_g is given by

$$V_g := \bigoplus_{\vec{i} \in \mathcal{I}^g} V_g^{\vec{i}} \tag{3}$$

summing over all possible g-tuples of simple objects.

TQFT and the spaces V_g

Assume $\forall i, j, k \in \mathcal{I}$ admissible, $\mathrm{Hom}(i, j \otimes k) \cong \mathbb{C}$. For each such Hom-set, we choose a generator and represent it by a graph:



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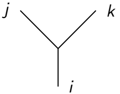

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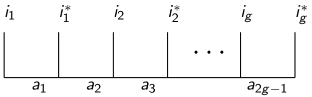
Then by definition, for $\vec{i} = (i_1, ..., i_g)$, the space $V_g^{\vec{i}}$ is spanned by the tree basis vectors

TQFT and the spaces V_g



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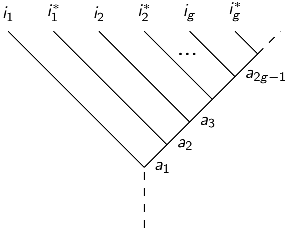

, (5)

where $\forall k \in \{1, ..., 2g-1\}, a_k \in \mathcal{I}$, and a_k can be obtained by fusing the vertical i-object on its right hand side and a_{k+1}.

The mapping class group representation ρ_g

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More explicitly, given a homeomorphism f in Γ_g, the matrix entry $\rho_g(f)_{T,T'}$ correponding to the tree basis vectors $T \in V_g^{\vec{i}}$ and $T' \in V_g^{\vec{i}'}$ can be computed as follows:

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- find a tangle presentation of f, denoted by $Tgl(f)$ (via surgery theory);
- extend the coloring of T to the bottom strands of $Tgl(f)$, and T' to the top strands of $Tgl(f)$;

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- for each coloring λ of the internal components of $Tgl(f)$, we get a morphism $Tgl(f)_\lambda$ in the MTC, let d_λ be the product of the quantum dimensions of the colorings in λ;

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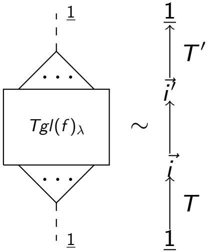
- Finally, evaluate the following diagram using graphical calculus, we get the desired matrix entry:

$$\rho_g(f)_{T,T'} = \mathcal{N} \sum_{\lambda} d_\lambda$$



$$\in \mathrm{Hom}(\underline{1}, \underline{1}) = \mathbb{C}. \tag{7}$$

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Let A_p, B_p, C_p be the right-handed Dehn twists along the p-th α-, β- and waist curves, the maps $\{T_p, S_p\}_{p=1,\dots g} \cup \{D_q\}_{q=1,\dots,g-1}$ generate Γ_g, where

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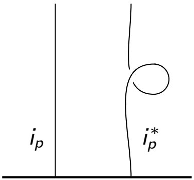
$T_p =$



, (9)

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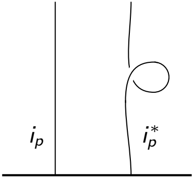
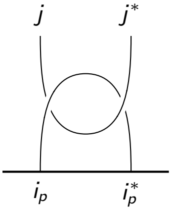
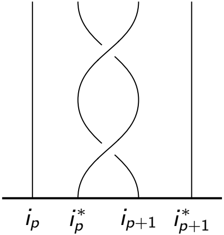
$T_p =$  , (9)

$S_p = \sum_j \frac{d_j}{D}$  , (10)

The mapping class group representation ρ_g

and

$D_p =$  . (11)

Definition of MMC

Definition. A *metaplectic modular category* of rank $(r+4)$ is a unitary modular category with $\mathcal{I} = \{\underline{1}, Z, Y_j, 1 \leq j \leq r, X, X'\}$ and the following fusion rules: let $m = 2r+1$,

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$$\begin{aligned}
X \otimes X &\cong \underline{1} \oplus \textstyle\bigoplus_{j=1}^{r} Y_j \\
X \otimes Y_j &\cong X \oplus X', \ 1 \leq j \leq r \\
X \otimes X' &\cong \underline{1} \oplus Z \oplus \textstyle\bigoplus_{j=1}^{r} Y_j \\
Z \otimes X &\cong X' \\
Z \otimes Z &\cong \underline{1} \\
Z \otimes Y_j &\cong Y_j, \ 1 \leq j \leq r \\
Y_j \otimes Y_j &\cong \underline{1} \oplus Z \oplus Y_{\min\{2j, m-2j\}}, \ 1 \leq j \leq r \\
Y_i \otimes Y_j &\cong Y_{|i-j|} \oplus Y_{\min\{i+j, m-i-j\}}, \ 1 \leq i, \ j \leq r, \ i \neq j.
\end{aligned} \tag{12}$$

Example: $\mathrm{SO}(m)_2$

Let $m = 2r + 1$, and $\mathfrak{g} = \mathfrak{so}(m)$, the representation theory of the quantum group $\mathrm{U}_q(\mathfrak{g})$ at $q = e^{\pi i/2m}$ gives rise to an MMC with the following S-matrix:

$$S = \begin{pmatrix} \frac{1}{2\sqrt{m}} & \frac{1}{2\sqrt{m}} & \frac{1}{\sqrt{m}} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2\sqrt{m}} & \frac{1}{2\sqrt{m}} & \frac{1}{\sqrt{m}} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{m}} & \frac{1}{\sqrt{m}} & H & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & 0 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \tag{13}$$

where H is understood as an $r \times r$-matrix with entries $H_{i,j} = 2\cos(2\pi ij/m)/\sqrt{m}$. We will call them the $\mathrm{SO}(m)_2$-theory.

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Goal: calulate ρ_g for $\mathrm{SO}(m)_2$ and discover interesting properties. Mainly focus on $\mathrm{SO}(5)_2$. Below is some data of the theory.

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$$\theta_Z = 1,\ \theta_X = e^{\pi \mathrm{i}/4},\ \theta_{Y_j} = e^{\frac{\pi \mathrm{i} j(m-j)}{m}},\ 1 \leq j \leq r; \qquad (14)$$

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some braidings (R-matrices):

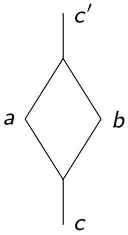
$$R_{\underline{1}}^{Y_1,Y_1} = e^{\frac{\pi \mathrm{i}(m-1)}{m}},\ R_Z^{Y_1,Y_1} = e^{\frac{-\pi \mathrm{i}}{m}}, \tag{15}$$

an example of F-matrix:

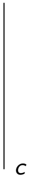
$$F_X^{Y_1 Y_1 X} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \tag{16}$$

Method: Graphical Calculus

By the guideline provided before, we need rules to perform graphical calculus, some of which are listed here, the colorings are all simple objects:



$$= \frac{\theta(a,b,c)}{d_c}\delta_{c,c'}$$



, (17)

where $\delta_{c,c'}$ is the Kronecker delta function, and $\theta(a,b,c) = \sqrt{d_a d_b d_c}$.

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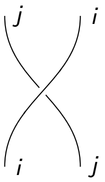
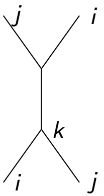


$$= \sum_k \frac{d_k R_k^{ji}}{\theta(i,j,k)} \qquad (18)$$



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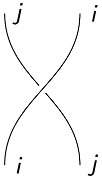

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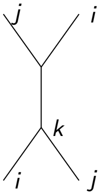

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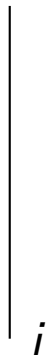

, (19)

Result

Theorem (W.)

Applying graphical calculus and the given data of $\mathrm{SO}(5)_2$*-theory, we completely determined* ρ_g *for every genus* g*.*

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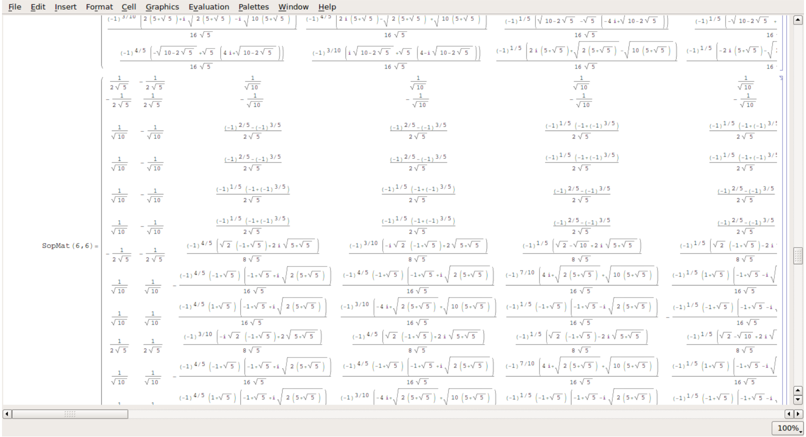
Applying graphical calculus and the given data of $\mathrm{SO}(5)_2$-theory, we completely determined ρ_g for every genus g.

However, as the dimensions of V_g are very large, it is impossible to present all the results here. Therefore, we give the result for $g = 1$ and some observations and partial results on higher genus.

Result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SopMat (6,6) =
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Result

Example. Note that when $g = 1$, Γ_g is generated by two elements S_1 and T_1, and that associated to the $\mathrm{SO}(5)_2$-theory, $\dim V_1 = 6$.The representation $\rho_1 : \Gamma_1 \to \mathrm{End}(V_1)$ associated to the $\mathrm{SO}(5)_2$-theory is given by:

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$$\rho_1(S_1) = \begin{pmatrix} \frac{1}{2\sqrt{5}} & \frac{1}{2\sqrt{5}} & \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2\sqrt{5}} & \frac{1}{2\sqrt{5}} & \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \frac{-1+\sqrt{5}}{2\sqrt{5}} & \frac{-1-\sqrt{5}}{2\sqrt{5}} & 0 & 0 \\ \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \frac{-1-\sqrt{5}}{2\sqrt{5}} & \frac{-1+\sqrt{5}}{2\sqrt{5}} & 0 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 & 0 & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & 0 & 0 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}, \qquad (20)$$

and

$$\rho_1(T_1) = diag(1, 1, e^{\frac{-4\pi \mathrm{i}}{5}}, e^{\frac{4\pi \mathrm{i}}{5}}, -\mathrm{i}, \mathrm{i}). \qquad (21)$$

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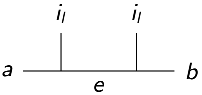
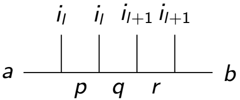


$: \ e \text{ admissible}\}, \qquad (22)$

and D_l preserves subspaces in the form of

$W^{\vec{i},l}_{a,b,p,r} := \mathrm{span}\{$



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Hence it suffices to determine the actions of the generators of Γ_g on these subspaces and then write out the representation in diagonal block matrix form.

Here we give an example of the action of D_2 on one of the subspaces $W = W_{Z,Z,Y_1,Y_2}^{(Z,Y_1,Y_2,Z),2}$ of V_4:

$$\rho_4(D_2)|_W = \begin{pmatrix} \frac{1}{2}e^{\pi i/5}(-1+e^{3\pi i/5}) & -\frac{1}{2}e^{\pi i/5}(1+e^{3\pi i/5}) \\ -\frac{1}{2}e^{\pi i/5}(1+e^{3\pi i/5}) & \frac{1}{2}e^{\pi i/5}(-1+e^{3\pi i/5}) \end{pmatrix}. \tag{24}$$

Eigenvalues

A direct computation confirms a more general argument on the eigenvalues of ρ_g associated to classical quantums groups at roots of unity (although I haven't seen it written down explicity in the literature):

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Corollary (W.)

The eigenvalues of $\rho_g(S_p), \rho_g(T_p), \rho_g(D_P)$ associated to $\mathrm{SO}(5)_2$ are 20-th roots of unity for all p.

Integrality

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At first, we found out by direct computation that:

Integrality

It is long known that for modular categories, TQFT constructions can be made over a cyclotomic field $\mathbb{Q}(\zeta) \subset \mathbb{C}$ for some roots of unity ζ. Furthermore, Gilmer-Masbaum-van Wamelen proved that the $SO(3)$-TQFT (different from the SO in our case) can be defined over $\mathbb{Z}[\zeta, \mathrm{i}]$.

Question: can we define our $\mathrm{SO}(m)_2$-TQFT over some ring of cyclotomic integers? Or, can we at least make some changes of bases so that image of ρ_g is over cyclotomic integers?

At first, we found out by direct computation that:

Theorem (Kerler, W.)

Under a change of basis, the images of $\rho_1(S_1)$ and $\rho_1(T_1)$ has entries in $\mathbb{Z}[\zeta]$ where $\zeta = e^{\frac{\pi \mathrm{i}}{5}}$ is a 10-th root of unity.

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For $m = 7, 11, 19$, ρ_1 *associated to* $\mathrm{SO}(m)_2$ *can be defined over* $\mathbb{Z}[\zeta_m, \mathrm{i}]$, *and for* $m = 13, 17$, *the corresponding* ρ_1 *can be defined over* $\mathbb{Z}[\zeta_m]$, *where* $\zeta_m = e^{2\pi \mathrm{i}/m}$.

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And we are optimistic to propose the following conjecture:

Integrality

Conjecture (W.)

Let m be an odd prime. The ρ_1 associated to $\mathrm{SO}(m)_2$ can be defined over $\mathcal{O}$, where

$$\mathcal{O} = \begin{cases} \mathbb{Z}[\zeta_m, i], & \text{if } m \equiv 3 \pmod 4 \\ \mathbb{Z}[\zeta_m], & \text{if } m \equiv 1 \pmod 4 \end{cases}. \tag{25}$$

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If you have any suggestions on how to attack this problem, we can work together!

Thank You!